Robustness Radius for Chamberlin-Courant on Restricted Domains

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Presented by: Prof. Henning Fernau

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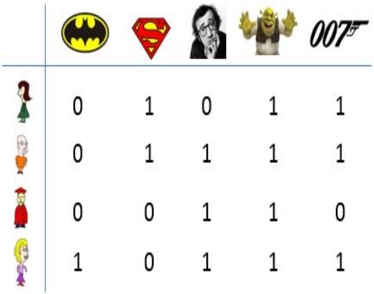
  where E represents election $E = (C, V)$ and $\mathcal{R}(E, k)$ is the **family** of k-sized subsets of C.
- For this work, we consider *Chamberlin Courant* voting rule.

Example:
Finding a collection of movies to include in Airplane

$n = 4, m = 5, k = 2$

Figure: Complete Orderings



0	1	0	1	1
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Figure: Approval Ballots

Note: Figures are taken from [1][1]

[1] Piotr Faliszewski: rapc-session-3-committees.pptx

Robustness Radius

- **Definition:** For multiwinner voting rule $\mathcal{R}$, and input $E = (C, V)$, a committee k and an integer r, we ask if it is possible to obtain an election E' by making *at most r swaps of adjacent candidates* within rankings of E (or change at most $k-$bits for the case of approval ballots) s.t. $\mathcal{R}(E, k) \neq \mathcal{R}(E, k)$

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  - YES $\rightarrow RR \leqslant r$
  - NO $\rightarrow RR > r$

Related Work

- 'Robustness Among Multiwinner Voting Rules' [SAGT'18] [2] – First defined the concept of **RR**

[2] Bredereck, R., Faliszewski, P., Kaczmarczyk, A., Niedermeier, R., Skowron, P., & Talmon, N. (2017, September). Robustness among multiwinner voting rules. In International Symposium on Algorithmic Game Theory (pp. 80-92). Springer, Cham.

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- We consider the exact algorithms for hard instances and ask the question on restricted domain for **CC** rule

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Similar Notions

- **RR** captures the impact of small changes in the input preferences on the set of the winning committees for given voting rule.

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- **Swap Bribery** – Also cares about the outcome after the change in profile [3].

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Preliminaries

- **Misrepresentation function:** For an m-candidate election with votes specified as complete order over set of candidates, a *dissatisfaction function* is given by a non-decreasing function $\alpha\colon [m] \to \mathbb{N}$ with $\alpha(1) = 0$.

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  - $\ell_1 = \sum_{i=1,\ldots,n} \alpha(\mathrm{pos}_{v_i}(\Phi(v_i)))$, and
  - $\ell_\infty(\Phi) = \max_{i=1,\ldots,n} \alpha(\mathrm{pos}_{v_i}(\Phi(v_i)))$

Chamberlin Courant Example:

0	4	1	2	3
2	3	0	1	4
2	1	4	3	0
1	0	2	4	3

Consider the committee: 

(considering ℓ_1 dis-satisfaction) Note: Figure is taken from [1][4]

[4] Piotr Faliszewski: rapc-session-3-committees.pptx

Chamberlin-Courant Voting Rule

Chamberlin Courant-rule

For given dissatisfaction function (α) and aggregation function (ℓ), the α-ℓ-*CC voting rule* is a mapping that takes an election $E = (C, V)$ and a positive integer k with $k \leqslant |C|$ as its input, and returns a k-CC-assignment function Φ for E that minimizes $\ell(\Phi)$. [5] [6]

[5] Chamberlin & Courant: Representative deliberations and representative decisions

[6] Betzler, Slinko & Uhlmann: On the Computation of Fully Proportional Representation

Parameterized Complexity

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- XP $\rightarrow$ if $\exists$ algorithm that decides in time $\mathcal{O}(n^{f(k)})$

Results Summary

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Note: All results are parameterized by parameter k- the size of the committee.

- For general profiles:
  1. RR is **W[2]**-hard for *Approval Chamberlin-Courant*.
  2. **XP** algorithm for determining **RR** for *complete rankings/ approval ballots*.

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- For general profiles:
  1. RR is **W[2]**-hard for *Approval Chamberlin-Courant*.
  2. **XP** algorithm for determining **RR** for *complete rankings/ approval ballots*.
- On nearly restricted domain:
  1. **RR** is **NP**-hard for $\ell_1 - CC$ even for *6-crossing* domains.
  2. **RR** is **NP**-hard for $\ell_\infty - CC$ even for *4-crossing* domains.
  3. **RR** is **NP**-hard for $\ell_\infty - CC$ even for *4-composite SP* domains.

Hardness for Approval Ballots

Theorem

Checking if **RR=1** for Approval Chamberlin Courant is **W[2]** hard parameterized by size of committee (k).

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- Reduction Hitting Set instance.

Hitting Set Instance:

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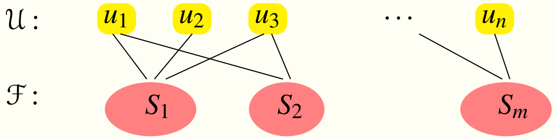

$\forall S_i \in \mathcal{F}; S_i \subseteq \mathcal{U}$

Given k, does there exist $S \subseteq U$ s.t. $|S \cap S_i| \neq \phi$ & $|S| \leqslant k$

Hitting Set
Example

Hitting Set Instance:

$\mathcal{U} = \{1, 2, 3, 4\}$

$\mathcal{F} = \{(1, 2, 3), (1, 2), (3, 4), (2, 4), (1, 4)\}$

$k = 1$

NO instance

Hitting Set
Example

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$\mathcal{U} = \{1, 2, 3, 4\}$

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$k = 2$

$S = \{1, 2\}$

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$|\mathcal{U}| = n$

$k = 2$ (size of the hitting set)

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RR Instance for Approval Voting Rule:

$k' = k$ (Committee size)

$\mathcal{A} := \underbrace{\{c_1, c_2, \ldots, c_n\}}_{\mathcal{U}} \cup \underbrace{\{d_1, d_2, \ldots, d_k\}}_{\mathcal{D}}$

Construction

Hitting Set Instance:
$\mathcal{U} = \{1, 2, 3, 4\}$
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$|\mathcal{U}| = n = 4$
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v_5	:	1	0	0	1	1	0

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Equivalence of two instances:

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  - 'YES' instance of **HS**
    $\implies$ at least 2, 2-sized winning committees
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  - Hence RR=1.

Equivalence of two instances: Reverse Direction

If there is more than one winning committee in the constructed election, then there exists a hitting set of size at most k.

If RR $= 1$ for the constructed election, then there are at least two winning committees of size k.

$$\Downarrow$$

If RR $= 1$ for the constructed election, the instance of HS on which the election is based is a YES-instance.

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If there is more than one winning committee in the constructed election, then there exists a hitting set of size at most k.

If there is exactly one winning committee in the constructed election, then any other committee has a dissatisfaction score of at least two.

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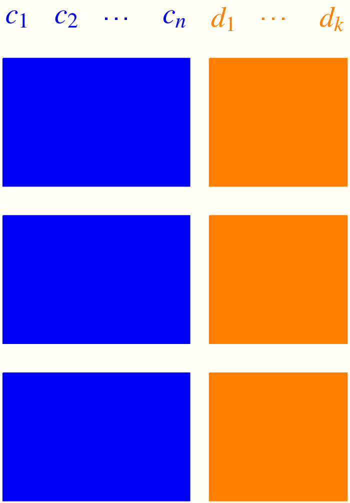
More than one winning committee



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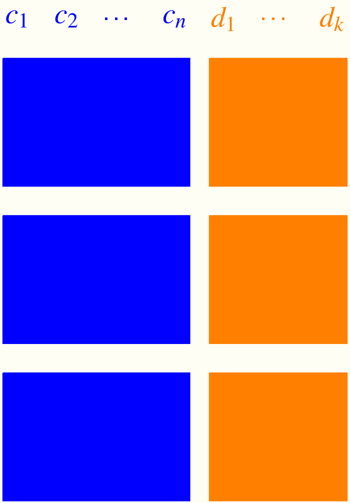

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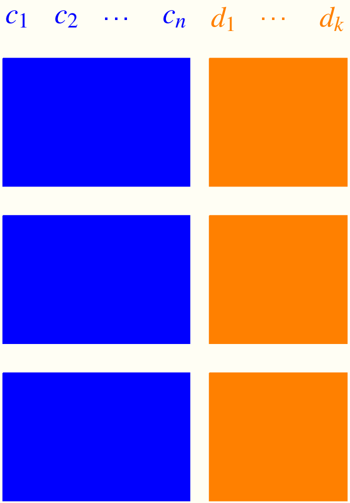

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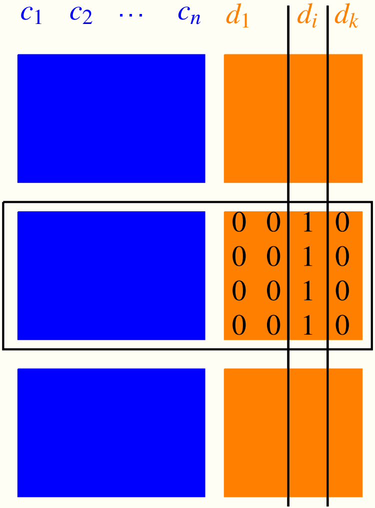

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Consider block corresponding to d_i.

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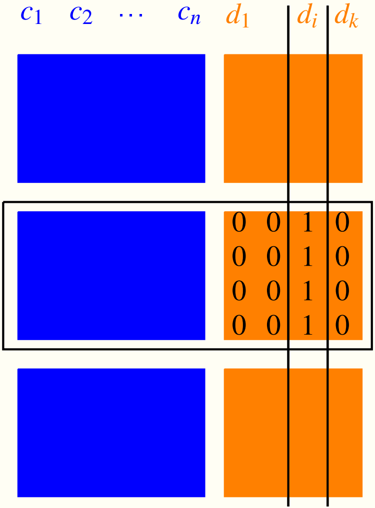

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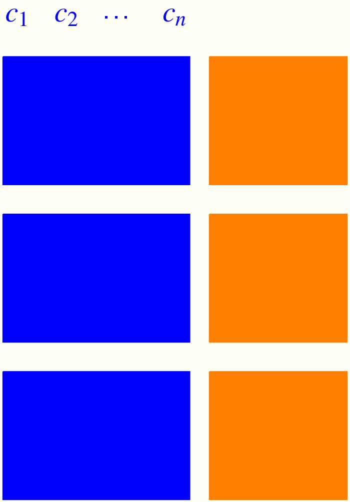
Here, every voter (set) is represented by a non-dummy candidate. Hence, $W \setminus D$ corresponds to a hitting set.

Case II

Unique winning committee



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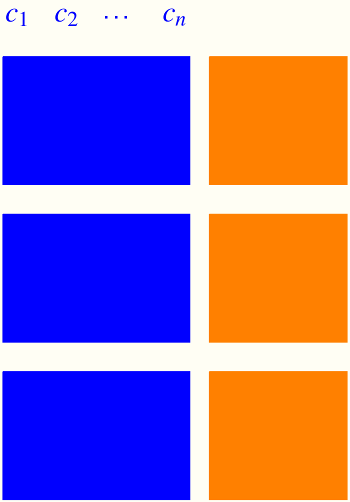

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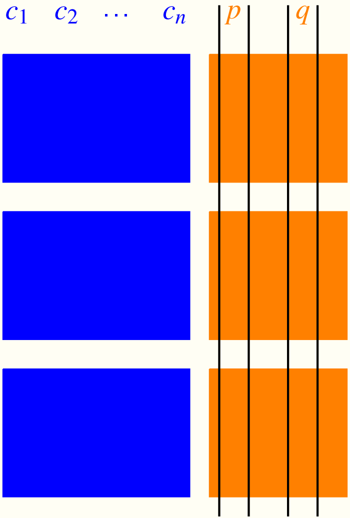

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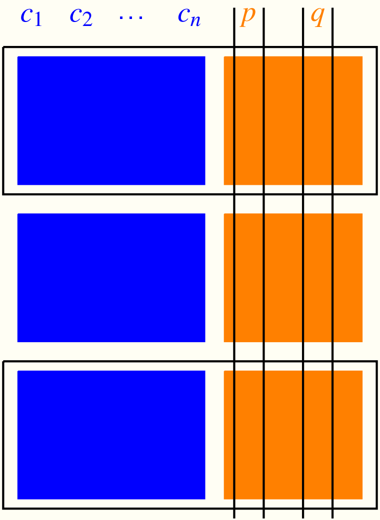

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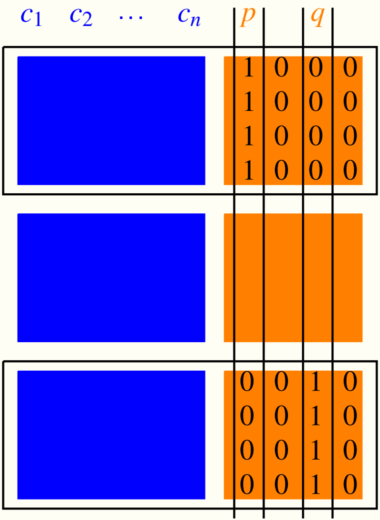

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Since $W \setminus D$ is not a hitting set*, there is at least one voter in each block that is not represented by W in each block.

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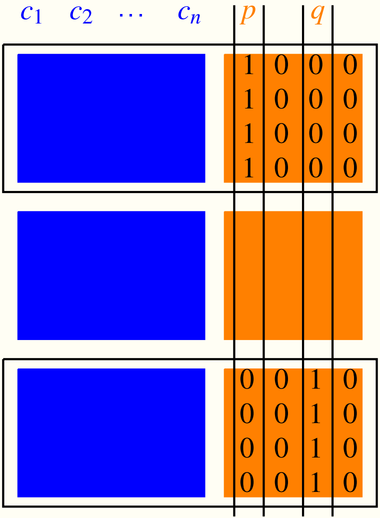
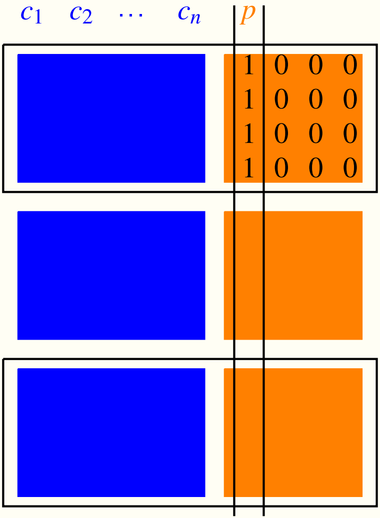


Suppose D is the only committee that represents every voter.
Let W be any other committee s.t. $|D| = |W|$.

Suppose W omits two candidates from D, say p and q.
Consider the voter blocks corresponding to p and q.

Since $W \setminus D$ is not a hitting set*, there is at least one voter in each block that is not represented by W in each block.
Hence the dissatisfaction of W is at least 2. (done)

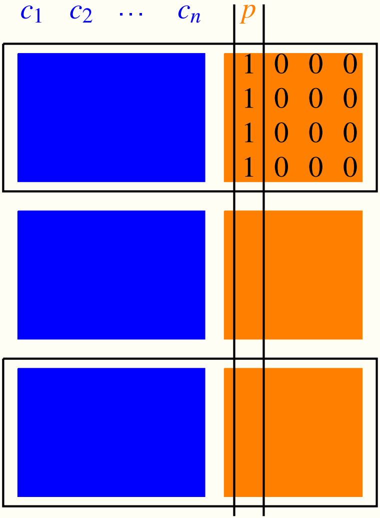
Case II
Unique winning committee



Suppose D is the only committee that represents every voter.
Let W be any other committee s.t. $|D| = |W|$.

Suppose W omits one candidate from D, say p.

Case II
Unique winning committee

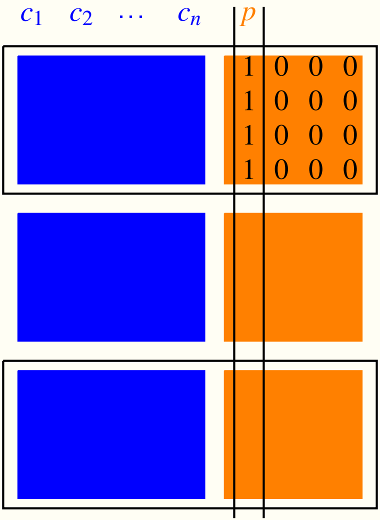


Suppose D is the only committee that represents every voter.
Let W be any other committee s.t. $|D| = |W|$.

Suppose W omits one candidate from D, say p.

Consider the block corresponding to p

Case II
Unique winning committee

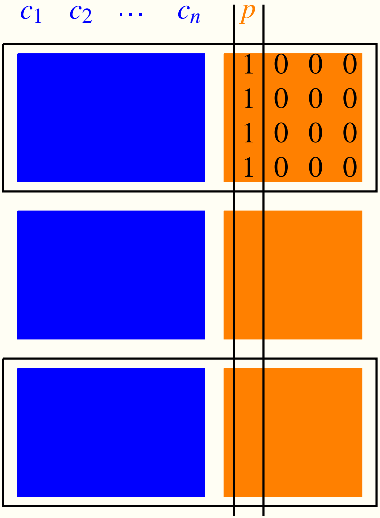


Suppose D is the only committee that represents every voter.
Let W be any other committee s.t. $|D| = |W|$.

Suppose W omits one candidate from D, say p.

Since $W \setminus D$ is not a hitting set*, there is at least one voter that is not represented by W is $p's$ block.

Case II
Unique winning committee



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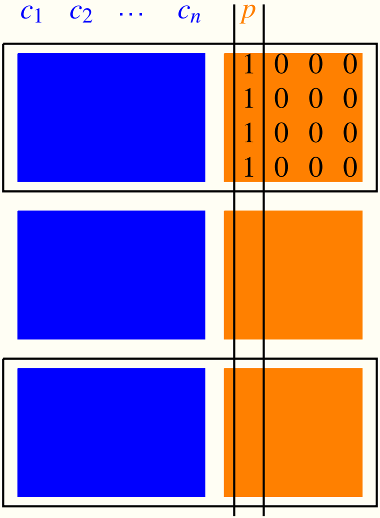
Case II
Unique winning committee



Suppose D is the only committee that represents every voter.
Let W be any other committee s.t. $|D| = |W|$.

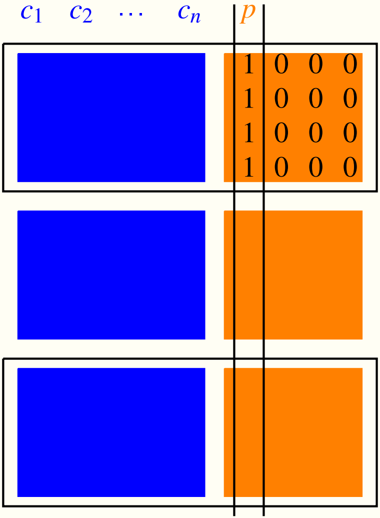
Suppose W omits one candidate from D, say p.

Since $W \setminus D$ is not a hitting set*, there are at least two voters that is not represented by W is $p's$ block. (Sub-case (a))

Hence, $dissatisfaction(W) > 1$.

Case II

Unique winning committee

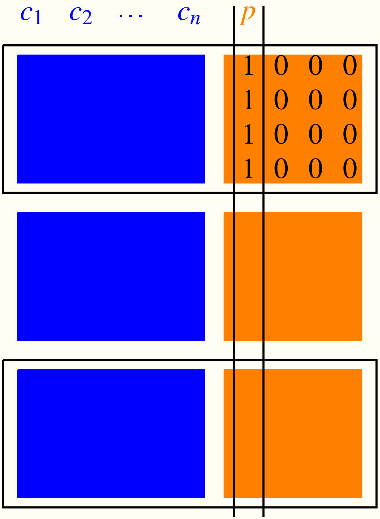


Suppose D is the only committee that represents every voter.
Let W be any other committee s.t. $|D| = |W|$.

Suppose W omits one candidate from D, say p.

Since $W \setminus D$ is not a hitting set*, there is exactly one voter v that is not represented by W is $p's$ block. (Sub-case (b))

Case II
Unique winning committee



Suppose D is the only committee that represents every voter.
Let W be any other committee s.t. $|D| = |W|$.

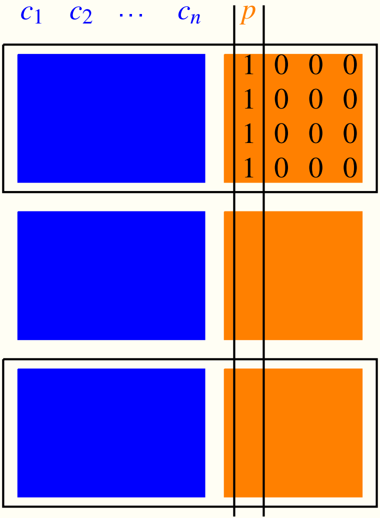
Suppose W omits one candidate from D, say p.

Since $W \setminus D$ is not a hitting set*, there is exactly one voter v that is not represented by W is $p's$ block. (Sub-case (b))

$W \setminus D$ combined with any element from the set corresponding to v, gives a hitting set of size at most k.

Case II

Unique winning committee



(Contradicts this assumption)
Suppose D is the only committee that represents every voter.
Let W be any other committee s.t. $|D| = |W|$.

Suppose D is the only committee that represents every voter.

Since $W \setminus D$ is not a hitting set*, there is exactly one voter v that is not represented by W is $p's$ block.

$W \setminus D$ combined with any element from the set corresponding to v, gives a hitting set of size at most k.

Thank You !