On the Complexity of Winner Verification and Candidate Winner for Multiwinner Voting Rules

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$v_2 : d$	$\succ b$	$\succ c$	$\succ a$
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Let committee $C_1 \leftarrow \{a,d\}$

Each voter is assigned to its favorite candidate in C_1

$$d(C_1) = 0+0+0+0 = 0$$

C_1 belongs to the set of winning committees if it achieves least dissatisfaction score

Monroe Voting Rule

$\mathcal{C}$ (Set of candidates) : $\{a, b, c, d\}$

$\mathcal{V}$ (Set of votes) :

0	1	2	3

$v_1 : a \succ b \succ c \succ d$

$v_2 : d \succ b \succ c \succ a$

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$v_4 : a \succ b \succ c \succ d$

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Input: $(\mathcal{C}, \mathcal{V}, k)$

Let $C_2 \leftarrow \{a, b\}$

Number of votes assigned to each candidate in C_2 is equal

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$$d(C_2) = 2$$

$C_2 \leftarrow$ a winning committee under Monroe rule

Motivation and Related Work

- Multiwinner elections are ubiquitous
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  E.g., choosing a governing body, airline movie selection
- CC and Monroe are designed to achieve the desirable property of Proportional representation [CC83] [M95]
- For both CC and Monroe, finding a fixed size committee with bounded dissatisfaction score are NP-complete [PRZ08] in the setting of rankings as well as approval ballots

Our Results

We study two natural variants of the winner determination problem

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Preferences	CC		Monroe	
	ℓ_1	ℓ_∞	ℓ_1	ℓ_∞
Ranking	?	?	?	?
Approval	?	-	?	-

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	ℓ_1	ℓ_∞	ℓ_1	ℓ_∞
Ranking	coNP	coNP	coNP	coNP
Approval	coNP	-	coNP	-

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Preferences	CC		Monroe	
	ℓ_1	ℓ_∞	ℓ_1	ℓ_∞
Ranking	θ_2^P [1]	θ_2^P	θ_2^P	θ_2^P
Approval	θ_2^P	-	θ_2^P	-

[1] The result was independently shown by [BFKNST19]

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- For CC, we show both Winner Verification and Candidate Winner problems are efficiently solvable on single-peaked domains
- We extend our results for single-crossing domains

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- Heuristics for both WV and CV