Equitable Division of a Path

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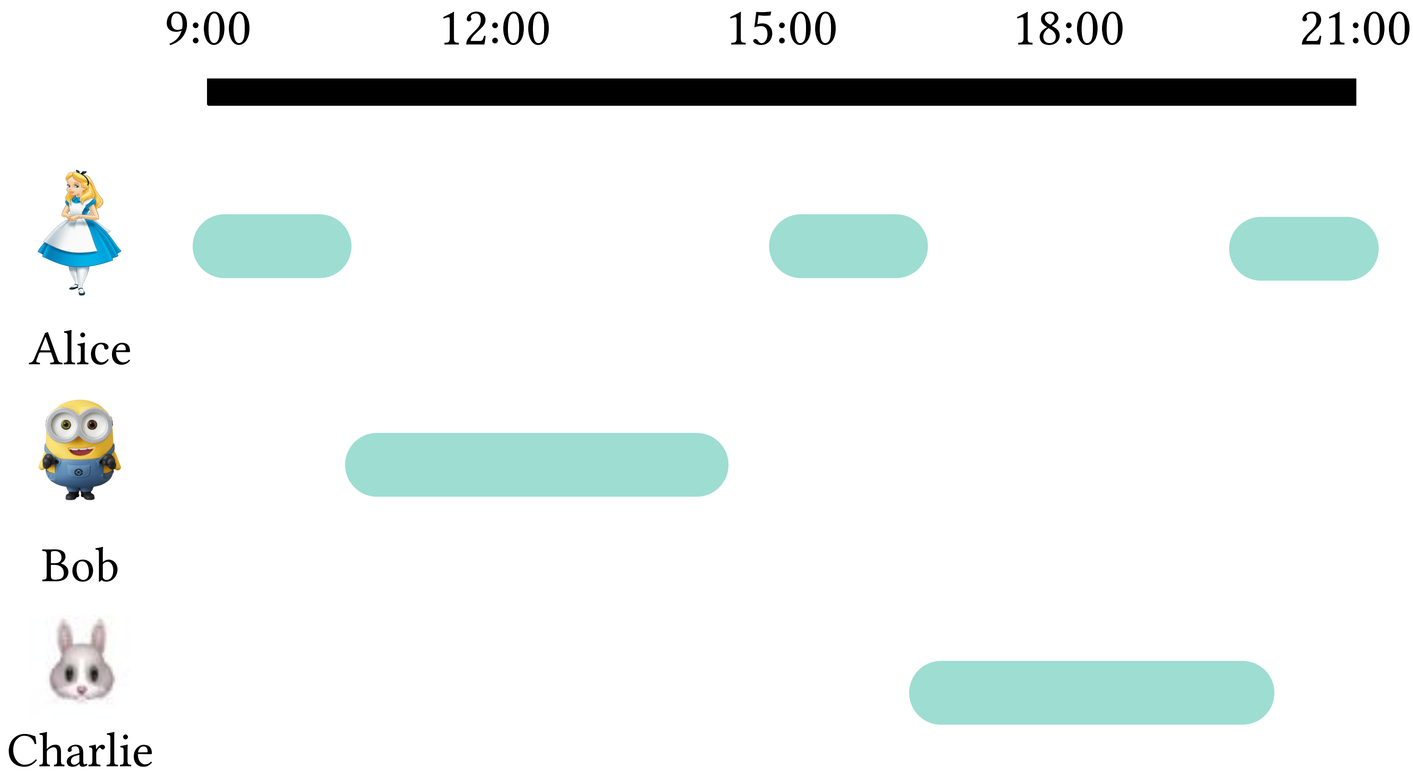
Joint work with Neeldhara Misra (IITGN), Rohit Vaish (TIFR), P. R. Vaidyanathan (TU Wien)

Motivation

Fair division of supercomputer access time



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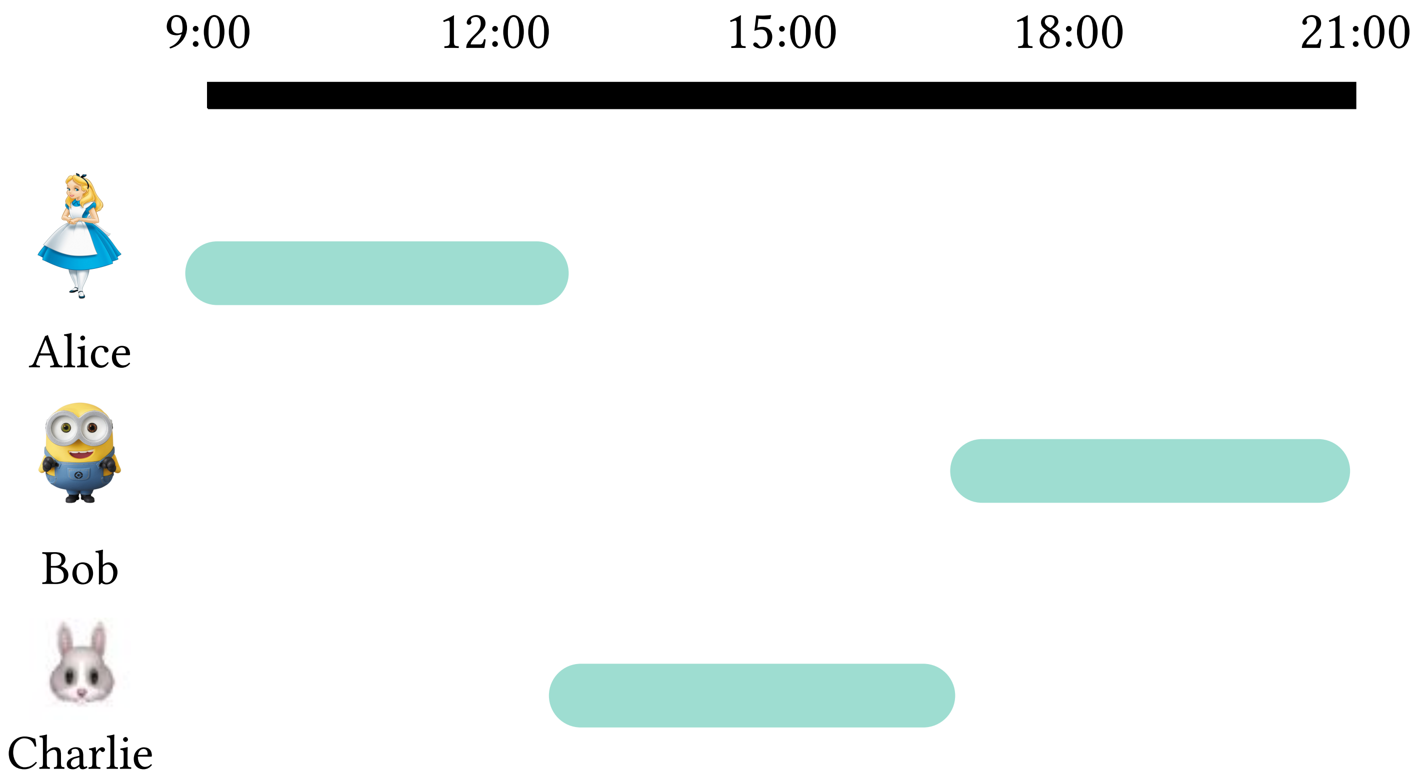
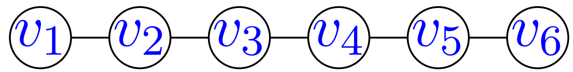


The Model



Alice	1	2	1	1	0	0
Bob	0	1	2	1	4	4
Charlie	0	0	2	0	4	2

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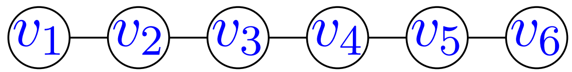

	v_1	v_2	v_3	v_4	v_5	v_6
Alice	1	2	1	1	0	0
Bob	0	1	2	1	4	4
Charlie	0	0	2	0	4	2

- Set of goods on a path
- Agents have additive cardinal preferences over goods

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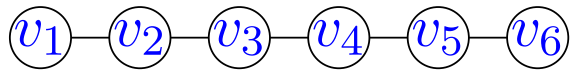

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Fairness Notions - Envyfreeness

- **Envyfreeness**: $u_i(A_i) \geq u_i(A_j)$ for each pair i, j

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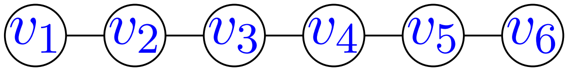

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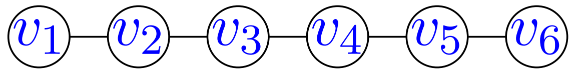

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Alice	1	2	1	1	0	0
Bob	0	1	2	1	2	4
Charlie	0	0	1	0	5	4

Except for v_6 my bundle is better than Charli's

- **Envyfreeness**: $u_i(A_i) \geq u_i(A_j)$ for each pair i, j
- **EF1**: $u_i(A_i) \geq u_i(A_j \setminus \{v\})$

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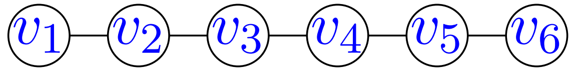

	v_1	v_2	v_3	v_4	v_5	v_6
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- **EF1**: $u_i(A_i) \geq u_i(A_j \setminus \{v\})$
- EF1 allocation is known to always exists for at most 4 agents [Bilò et al.'19]

Fairness Notions - Equitability

- **Equitability**: $u_i(A_i) \geq u_j(A_j)$ for each pair i, j

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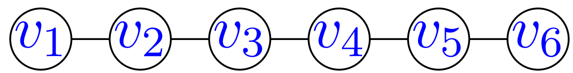

Alice	**1**	**2**	1	1	0	0	$u(A)$=3
Bob	0	1	**2**	**1**	**0**	4	$u(B)$=3
Charlie	0	0	1	0	4	**3**	$u(C)$=3

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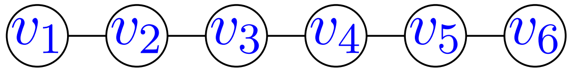
Fairness Notions - Equitability



Alice	1	2	1	1	0	0	$u(A)$=3
Bob	0	1	2	1	2	4	$u(B)$=3
Charlie	0	0	1	0	5	2	$u(C)$=7

Except for v_5 my bundle is better than Charli's

- **Equitability**: $u_i(A_i) \geq u_j(A_j)$ for each pair i, j
- **EQ1**: $u_i(A_i) \geq u_j(A_j \setminus \{v\})$ [Freeman et al.'19]

Do connected EQ1 allocations always exist?

Theorem

A connected, complete, EQ1 allocation of a path can be computed in a polynomial time.

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Theorem

A connected, complete, EQ1 allocation of a path can be computed in a polynomial time.

- ✓ Algorithm computes such an allocation consistent with any given left to right ordering of the agents
- ✓ Egalitarian-optimal allocation for the given ordering
- ✓ Extends to the case of monotonic valuations and chores

What about EQ1+PO allocations?

Pareto Optimality: Any redistribution making an agent better off makes at least one agent worse off

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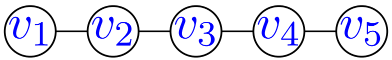
- EQ1+PO allocation do not always exist:



	v_1	v_2	v_3	v_4	v_5
Alice	1	0	0	0	1
Bob	0	1	1	1	0

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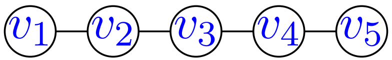

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Theorem

Checking the existence of a connected allocation that is EQ1 + PO is NP-complete.

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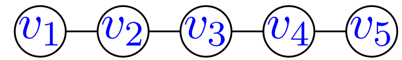


	v_1	v_2	v_3	v_4	v_5
Alice	1	0	0	0	1
Bob	0	1	1	1	0

Theorem

Checking the existence of a connected allocation that is EQ1 + PO is NP-complete.

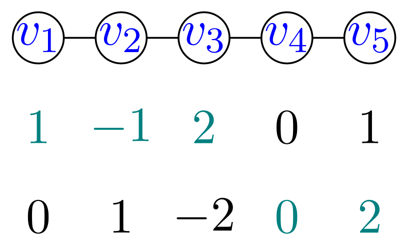
✓ Hardness holds even on sparse instances

✓ Strengthens the known hardness result for EF1+PO allocations [IP'19]

Summary & Open Problems

	EQ1+complete	EQ1+Pareto optimal	EQ1+Non-Wasteful
Existence	Yes	No	No
Computation	Polytime	NP-complete	NP-complete

- Extension to general graphs
- Goods and chores:



Restricted Domains

✓ Efficient algorithm to compute EQ1+NW allocation for extremal preferences